AD-A137 748 APPLICATIONS OF FUNCTIONAL ANALYTIC AND MARTINGALE METHODS TO PROBLEMS IN.. (U) MASSACHUSETTS UNIV AMHERST DEPT OF MATHEMATICS AND STATISTICS.. W A ROSENKRANTZ 1/1

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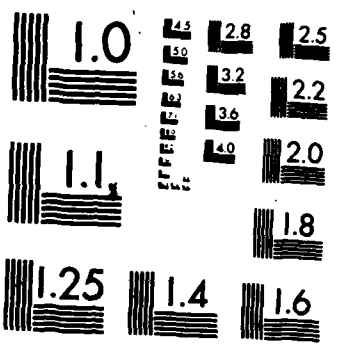

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

This report summarizes results and lists publications emerging from this research period in the area of performance analysis of random access communications systems.





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ITEM #11, TITLE: APPLICATIONS OF FUNCTIONAL ANALYTIC AND MARTINGALE METHODS TO PROBLEMS IN QUEUEING NETWORK THEORY



AFOSR-TR- 84 0030

Title: Applications of Functional Analytic and Martingale Methods to Problems in Queueing Network Theory -- Annual Scientific Report on Air Force Grant #82-0167

Principal Investigator: Professor Walter A. Rosenkrantz

I. Publications

(1) Calculation of the LaPlace transform of the length of the Busy period for the M/G/1 Queue via Martingales, AFOSR 82-0167, Report No. 1, Annals of Probability, Vol. II, No. 3, August, (to appear).

(2) Diffusion Approximation for a class of Markov processes satisfying a non-linear Fokker-Planck equation (with Li Zhan Bing), AFOSR 82-0167, Report No. 2, Journal of Non-linear Analysis, (to appear).

(3) On the Instability of the Slotted Aloha Multiaccess Algorithm AFOSR 82-0167, Report No. 3 (with Don Towsley), IEEE Transactions on Automatic Control (to appear).

(4) Weak Convergence of a Sequence of Queueing and Storage Processes to a Singular Diffusion, AFOSR 82-0167, Report No. 4.

II. In Preparation

(1) On the stability of the exponential back off Algorithm for an ETHERNET type access system.

(2) Weak convergence of a sequence of Markov processes to a Multivariate Ornstein-Uhlenbeck process (joint with F. Bennett).

III. Coupling Activities i.e. Lectures, Conferences, Symposia, etc.

(1) I attended the IEEE Information Theory Group International Symposium on Information Theory in Les Arcs, France, June 21-25, 1982. I presented the paper "Application of Functional Analytic and Martingale Methods to Markov processes occurring in Queueing Theory" in session D6. In addition to presenting the above paper I had the privilege of meeting and discussing problems in Stochastic processes and random access communications with Professors B. Hajek, A. Ephremides, E. Wong, F. Beutler, and R. Boel.

(2) I presented AFOSR 82-0167, Report No. 4 to the "International Seminar on Modelling and Performance Evaluation Methodology" held in Paris, January 24-26, 1983. During the conference I consulted with G. Fayolle of INRIA, S. Lavenberg IBM, P. Brémaud of ENSTA, R. Boel, A. Ephremides and A. Makowski of the Electrical Engineering Department of the University of Maryland, College Park. After the

conference I traveled to the Ecole Polythechnique to consult with Professor M. Métivier and a student of his (Mme de Zélincourt) who has obtained some interesting results on the Diffusion Approximation for Queueing systems via the Martingale Methods of Stroock-Varadhan.

(3) I accepted an invitation to present a lecture entitled "Some Applications of Martingale Theory to Queueing Theory" to the Colloquium series in the Department of Electrical Engineering of the University of Maryland, April 21, 1983.

(4) The principal investigator invited Professors George Papanicolaou of the Courant Institute and Stanley Sawyer of Purdue University to consult with him on mathematical problems that arose in the course of his research.

IV. Professional Personnel associated with the research effort.

(1) F. Bennett, PH.D. candidate in the Department of Mathematics and Statistics at the University of Massachusetts. Ms. Bennett has been working on a Ph.D. thesis in mathematics under my direction for the past two years and has been partially supported during the summer months. Tentative title for her thesis is "On a class of Multivariate Markov processes converging to Multivariate Ornstein Uhlenbeck process". Her problem is to justify an heuristic diffusion approximation due to McNelll and Schach. Technical difficulties arise from the fact that the drift terms of the limiting diffusion are unbounded but linear. Her problem then is to obtain a regularity theorem (or a core) for the limiting parabolic partial differential equation. She is now writing up one solution to this problem and I expect her to receive the Ph.D. degree in August of '83.

V. Status of the research.

The main thrust of the research has been and continues to be the performance analysis of random access communications systems. One of the most important problems is whether or not a protocol for accessing a communications channel is stable. In AFOSR 82-0167, Report No. 3 we showed that the SLOTTED ALOHA Multi access algorithm is strongly unstable i.e. transient via a Martingale method of independent interest. The idealized Ethernet system upon which we are now working is more challenging since the number of packets $y(t)$ that have been blocked at time $t$ no longer forms a Markov chain. Using Martingale theory we have been able to show when the arrival Process is Poisson (with arrival rate $\mu$ and retransmission probabilities $p_k = 2^{-k}$, $k=1,2,\ldots$) that the Ethernet system with an infinite number of users is unstable for $\mu \geq .72$. It has been conjectured that is is unstable for $\mu > 0$ but we have not been able to prove this. This problem is very interesting from the mathematical point ov view because it appears that one can model this

system as a complicated system of interacting particles with interactions that are unfortunately not of the "nearest neighbor" type. We intend to pursue this approach in discussions with colleagues at forthcoming AMS summer seminar at Boulder, Colorado.

In AFOSR 82-0167, Report No. 4 we studied a sequence of Queueing and Storage processes which converge weakly to a Bessel process with negative drift. Because of some non trivial technical difficulties we were unable to use the methods of Papanicolaou, Stroock and Varadhan. Exploiting a technique of Burman's as well as perturbation theorem of Kato we were able to use the classical Trotter-Kato approach to derive a diffusion approximation. One possible application of these ideas is to the study of the asymptotic behaviour of stable protocols when the arrival rate approaches its critical value.

Finally, in response to the AFOSR's initiative for basic research in reliability modeling and inference for realistic systems we have begun to study the Martingale approach to "Non parametric tests for comparison of counting Processes, with applications to Censored Survival Data" cf. paper by Andersen et.al. in the International Statistical Review 50(1982), pp 219-258. The idea here, briefly, is to apply the Doob-Meyer decomposition to the counting processes which occur in reliability theory and to exploit the Martingale calculus of Meyer, Ikeda-Watanabe to derive results, both old and new, on the asymptotic behaviour of various nonparametric tests within the unified framework of the Martingale Calculus.

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